

Qubits, superqubits and squbits

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Abstract

We analyze recently proposed formalisms which use nilpotent variables to describe and/or generalize qubits and notion of entanglement. There are two types of them distinguished by the commutativity and or anticommutativity of basics variables. While nilpotent commuting variables suit the new description of the qubits and entanglement, the anticommuting do not, but they can be used to describe generalized objects - superqubits. A squbit, in the present context, is a version of superqubit introduced to cure some of problematic properties of the superqubit.

1 Introduction

New algebraic approaches to study qubit systems and their generalizations in the context of quantum entanglement have appeared recently. Depending on the basics assumption of the variables realizing the two-levelness of the quantum system, the final outcome is different. The idea to describe the two level system by specially chosen variables is around for some time (see account and references in [1]) The basics assumption is realized by the condition

$$1 = (\Gamma)^0, \quad \Gamma, \quad (\Gamma)^2 = 0, \quad (1)$$

which yields the use of nilpotents with vanishing square (so called nilpotents of order one). Obviously one can consider higher order nilpotents to describe multi-level system e.g. qutrits [2]. Realization of nilpotency is very natural in the Grassmann algebra and variables of this type are used in formulation of supersymmetry since the late seventies of the Twentieth Century. Usually such variables are denoted by θ . Another possibility of realization of nilpotency, despite the graded commutative Grassmannian algebra, gives the commutative algebra with nilpotents: $\eta, \eta^2 = 0$. This option was used by physicists a bit later [3, 4, 2, 5, 6], firstly within the statistical physics to realize the Pauli exclusion principle for spinless objects like fluxes. Recently there was developed classical [5] and quantum [2, 6] formalism based on such variables to describe multiqubit systems. This formalism in natural way allows to study multiqubit entanglement and structure of state spaces.

It turns out that anticommuting θ variables and the superspaces are not well suited to study the multiqubit systems and the questions of entanglement.

In particular, natural realizations of supersymmetry concern rather different systems than qubits, moreover the conventional Berezinian/superdeterminant is not good for characterization of entanglement. It is worth noting that the first time the supersymmetrization of the qubit was proposed by Hruby [7], that was done within the supersymmetric field theoretical model where the qubit field has its anticommuting partner antiqubit. However there one uses rather field theoretical generalization of qubit to QFT-qubit. Here we are focused on nonrelativistic quantum mechanical objects.

Table 1: Comparison of the θ and η description of two qubit system.

Anticommutative	Commutative
$g(\theta_1, \theta_2) =$ $g_0 + \theta_1 g_1 + \theta_2 g_2 + \theta_1 \theta_2 g_{12}$	$f(\eta_1, \eta_2) =$ $f_0 + \eta_1 f_1 + \eta_2 f_2 + \eta_1 \eta_2 f_{12}$
$Berg = \begin{pmatrix} g_0 & g_1 \\ g_2 & g_{12} \end{pmatrix}$ $= (g_0 - g_1 g_{12}^{-1} g_2) g_{12}^{-1}$	$det f = \begin{pmatrix} f_0 & f_1 \\ f_2 & f_{12} \end{pmatrix}$ $= f_0 f_{12} - f_1 f_2$

Comparing formulas in Tab. 1 let us observe that expansion coefficients g_1 and g_2 of the superfunction g have to be odd elements of a graded algebra and cannot be reduced to the complex or real numbers. For η -function one can take all coefficients f_i , f_{12} complex or real valued. This point is essential from the physical point of view. Namely, when treating $g(\theta_i)$ and $f(\eta_i)$ as a generalized wave functions and developing generalization of the Schrödinger realization for θ variables one gets Grassmann algebra valued scalar product [8, 9, 10] and graded algebra valued "probability amplitudes". Therefore some additional mapping is needed to get numerical information from such a Grassmann element. Usually the so called *body mapping* [8] is used, other possibilities gives Banach-Grassmann algebra defined by Rogers [11, 12]. The η -functions do not yield such a complications. Such approaches were already considered in the context of supersymmetric quantum mechanics in super-Hilbert spaces cf. Refs. [13, 8, 9, 10]. Using super-Hilbert space approach one can consider e.g. coherent fermionic states which have properties analogous to the conventional coherent states, but are not physical in the usual sense. The supermatrix and matrix appearing in the Table 1 can be thought as a matrix of second order θ -superderivative and η -derivative. Again, in the case of the θ -function the properties of the Berezinian prevent us from getting information about factorization properties of the $g(\theta_1, \theta_2)$, while conventional determinant for the $f(\eta_1, \eta_2)$ does the job. In supersymmetric theories, but not only, the anticommuting variables are used to describe fermionic degrees of freedom. As was observed by Kitayev in the context of quantum information theory multi-fermion states are more non-local than bosonic ones, therefore we can expect that objects like superqubit may produce more nonlocality.

One can expect that superspaces related to graded commutative algebra might be not right arena to describe qubits, but as it was shown in Ref. [14]

specific supersymmetric generalization of qubit. Moreover for the superqubit systems the notion of entanglement has to be modified due to some algebraic artefacts. As it was noted above, from the physical point of view the use of anticommuting variables yields some difficulties, that is why there was attempt to cure some of them, and modify the notion to the so called squbits [15] - prefix s comes here from abbreviated prefix super. Let us note that the term “squbit” is already used in literature in quite different context. In the nanotechnology information technology it denotes superconducting qubit - a device based on Josephson’s junction.

In the following we shall sketch the description of qubits in terms of the η -variables, recall main properties of superqubits [14]. Then using results of Ref. [15] we will discuss briefly the modification of superqubit to the squbit.

Before going into details let us comment on chronology of the main concepts discussed in the present paper:

- 1976 - supersymmetric quantum mechanics introduced in 1976 by Nicolai [13]
- 1970 - origins of quantum computing, qubits
- 2006 - qubits described by commuting nilpotent variables [2, 1]
- 2009 - superqubits [14]
- 2010 - squbits [15]

2 Qubit - η -space description

The formalism of η -Hilbert spaces gives new tools to study multiqubit entanglement. Instead of considering the \mathbb{C}^2 Hilbert space and its tensor product structures one uses properties of functions of η -variables. Namely, if we take explicit form of one qubit and two qubit states in e.g. binary bases

$$\psi^{(1)}(x) = \psi_0(x)|0\rangle + \psi_1(x)|1\rangle, \quad (2)$$

$$\psi^{(2)}(x) = \psi_0(x)|00\rangle + \psi_1(x)|10\rangle + \psi_2(x)|01\rangle + \psi_{12}(x)|11\rangle, \quad (3)$$

they can be written as the η -variable functions as follows

$$\psi(x, \eta) = \psi_0(x) + \eta\psi_1(x) \quad (4)$$

and similarly for two qubits

$$\psi^{(2)}(x, \eta_1, \eta_2) = \psi_0(x) + \eta_1\psi_1(x) + \eta_2\psi_2(x) + \eta_1\eta_2\psi_{12}(x) \quad (5)$$

In the case of pure states it provides strong tool - functional determinants and criteria for factorability of η -functions. This information is in direct correspondence to the known measures of entanglement.

The formalism can be briefly presented as follows. The η variables are nilpotent elements coming from the commutative algebra \mathcal{N} generated by unit and the set of nilpotents of the first order. We can split given element to the numerical part and the rest $\mathcal{N} \ni \nu = b(\nu) + s(\nu)$ - the body and soul of an element. To describe the qubit and many-qubit states one takes bimodule over \mathcal{N} . \mathcal{H}

$\rightsquigarrow \mathcal{N}$ -module with the \mathcal{N} -scalar product i.e. The generalized scalar product is defined as

$$\langle ., . \rangle: \mathcal{H} \times \mathcal{H} \mapsto \mathcal{N} \quad (6)$$

such that for $F, G \in \mathcal{H}$

$$\nu^* \langle F, G \rangle = \langle \nu F, G \rangle = \langle F, \nu^* G \rangle, \quad \nu \in \mathcal{N} \quad (7)$$

$$\langle F, G \rangle = 0 \quad \forall G \in \mathcal{H} \Rightarrow F = 0 \quad (8)$$

$$b(\langle F, G \rangle)^* = b(\langle G, F \rangle) \quad (9)$$

$$b(\langle F, F \rangle) \geq 0, \quad \forall F \in \mathcal{H} \quad (10)$$

Particular realization of such \mathcal{N} -module is given by the function space $F[\vec{\eta}_n]$.

$$\langle F, G \rangle_{\mathcal{N}} = \int F^*(\vec{\eta}) G(\vec{\eta}) e^{\langle \vec{\eta}^*, \vec{\eta} \rangle} d\vec{\eta}^* d\vec{\eta}, = \int F^*(\vec{\eta}) G(\vec{\eta}) d\mu(\vec{\eta}^*, \vec{\eta}) \quad (11)$$

where

$$F^*(\vec{\eta}) = \sum_{k=0}^n \sum_{I_k} F_{I_k}^* \eta^{I_k^*}, \quad (12)$$

and $I_k = (i_1, i_2, \dots, i_k)$ is an ordered multi-index.

$$\langle F, G \rangle_{\mathcal{N}} = \sum_{k=0}^n \sum_{I_k} F_{I_k}^* G_{I_k} \quad (13)$$

Note that when $F_{I_k} \in \mathbb{C}$ the generalized \mathcal{N} -scalar product takes complex values and such assumption is natural for qubit's states description.

2.1 Entanglement of two qubits

To illustrate how factorization criteria compare to entanglement let us take two qubit η -function (5). The w_{12} denotes the η -Wronskian with respect to η_1 and η_2 variables

$$w_{12} = \det W_{12} = \begin{vmatrix} F & \frac{\partial F}{\partial \eta_1} \\ \frac{\partial F}{\partial \eta_2} & \frac{\partial^2 F}{\partial \eta_1 \partial \eta_2} \end{vmatrix} = \begin{vmatrix} F & \partial_1 F \\ \partial_2 F & \partial_{12} F \end{vmatrix} = F_0 F_{12} - F_1 F_2 \quad (14)$$

There is valid the following statement: for arbitrary function $F(\eta_1, \eta_2)$ vanishing of the Wronskian $w_{12}(F)$ is equivalent to factorization of the form $F(\eta_1, \eta_2) = G(\eta_1)\tilde{G}(\eta_2)$, for some G and \tilde{G} . Wronskian for the function of two η variables has numerical values. Considering the Werner state represented by the function $\psi_W(\eta_1, \eta_2) = \frac{1}{\sqrt{2}}(\eta_1 + \eta_2)$ we get that $w_{12}(\psi_W) = -\frac{1}{2}$. For the GHZ state $\psi_{GHZ}(\eta_1, \eta_2) = \frac{1}{\sqrt{2}}(1 + \eta_1 \eta_2)$ and $w_{12}(\psi_{GHZ}) = \frac{1}{2}$

The two-tangle expressed by the Wronskian takes the form

$$\tau(\psi) = 4|w_{12}(\psi)|, \quad (15)$$

e.g. $\tau(\psi_W) = 1$, $\tau(\psi_{GHZ}) = 1$. This approach works for multi-qubit states. For three qubits and more the Wronskians depend explicitly on η -variables, what yields the array of conditions and selection of important invariants for multi-qubit states. What is interesting the constraints coming from these criteria are desirable, because they select a subfamily of important invariants from the very quickly growing with the number of qubits, the set of all invariants. Detailed account of this approach can be found in the Refs. [16, 6].

3 Superqubit - extension of qubit to (2|1) super-space

The superspace formalism turns out to fit the description of the extension of qubit to the system which is symmetric with respect of the $Osp(1|2)$, the minimal supersymmetric extension of the $SL(2, \mathbb{C})$ which is the group SLOCC transformations playing the fundamental role in description of entanglement. This was the departure point in the definition of superqubit given in Ref. [14]. Let the $\mathcal{Q} = \mathcal{Q}_0 + \mathcal{Q}_1$ denotes the Banach-Grassmann algebra in the sense of Rogers, where \mathcal{Q}_i is its even and odd part for $i = 0$ and $i = 1$ respectively. The numerical part of $q \in \mathcal{Q}$ is called body the rest is a soul of the element q : $q = b(q) + s(q)$. One considers a graded module $\mathbb{V} = \mathbb{V}_0 \oplus \mathbb{V}_1$ of dimension (2|1) over the algebra \mathcal{Q} which extends the \mathbb{C}^2 Hilbert space of a qubit.

$$\mathbb{C}^2 \rightsquigarrow \mathbb{V}_{\mathcal{Q}}^{(2|1)} \quad (16)$$

with the \mathcal{Q} -scalar product i.e.

$$\langle \cdot, \cdot \rangle: \mathbb{V} \times \mathbb{V} \mapsto \mathcal{Q} \quad (17)$$

such that for $\psi, \phi \in \mathbb{V}$

$$\langle \psi, \phi q \rangle = \langle \psi, \phi \rangle q \quad q \in \mathcal{Q} \quad (18)$$

$$\langle \psi, \phi \rangle^\# = (-1)^{\psi(\phi+1)} \langle \phi, \psi \rangle, \quad (19)$$

where $(q^\#)^\# = (-1)^{|q|} q$ for homogenous elements i.e. $q \in \mathcal{Q}_i$, $|q| = i$, $i = 0, 1$.

The one-superqubit system is described by supervector

$$\psi = |0\rangle\psi_0 + |1\rangle\psi_1 + |\bullet\rangle\psi_\bullet = |\psi\rangle = |k\rangle\psi_k + |\bullet\rangle\psi_\bullet \equiv \sum_X |X\rangle\psi_X \quad (20)$$

where $|i\rangle$, $i = 0, 1$ are even vectors, $\psi_i \in \mathcal{Q}_0$ and $|\bullet\rangle$ is an odd vector, $\psi_\bullet \in \mathcal{Q}_1$. In this way the two level system is extended to the three level system with one level being of different nature then the other two. Moreover, there is no balance between odd and even (bose and fermi) degrees of freedom as in usual supersymmetric models. The \mathcal{Q} -scalar square $\langle \psi, \psi \rangle = \delta^{ij} \psi_i^\# \psi_j - \psi_\bullet^\# \psi_\bullet$ gives value with the nontrivial soul and normalized superstate has the form

$$|\psi\rangle = (\delta^{i_1 i_2} \psi_{i_1}^* \psi_{i_2})^{-\frac{1}{2}} \left(|k\rangle (1 + \frac{1}{2} (\delta^{i_1 i_2} \psi_{i_1}^* \psi_{i_2})^{-1} \psi_\bullet^\# \psi_\bullet) \psi_k + |\bullet\rangle \psi_\bullet \right) \quad (21)$$

3.1 Two superqubits

To see how questions concerning entanglement can be studied let us consider a two-superqubit states [14]

$$|\psi\rangle = |jk\rangle\psi_{jk} + |j\bullet\rangle\psi_{j\bullet} + |\bullet k\rangle\psi_{\bullet k} + |\bullet\bullet\rangle\psi_{\bullet\bullet} \quad (22)$$

$\dim \mathbb{V}^{(2)} = (5|4)$; ψ_{jk} , $\psi_{\bullet\bullet}$ are even and $\psi_{\bullet k}$, $\psi_{j\bullet}$ are odd. The super-SLOCC now is given by $Osp(2|1) \times Osp(2|1)$. To define the generalization of the entanglement for two-superqubits we can use the $(2|1) \times (2|1)$ -supermatrix

$$\psi_{XY} = \begin{pmatrix} \psi_{jk} & \psi_{j\bullet} \\ \psi_{\bullet k} & \psi_{\bullet\bullet} \end{pmatrix}. \quad (23)$$

Natural generalization of determinant is the Berezinian, but it is not good candidate for super-generalization of 2-tangle. Its explicit form

$$Ber(\psi_{XY}) = \det(\psi_{jk} - \psi_{j\bullet}\psi_{\bullet\bullet}^{-1}\psi_{\bullet k})\psi_{\bullet\bullet}^{-1} \quad (24)$$

shows that invertibility of $\psi_{\bullet\bullet}$ is too strong condition, restricting the possible two-superqubit states, since for example for the product state the $\psi_{\bullet\bullet}$ will be a product of two odd Grassmann numbers and therefore noninvertible. The solution given in Ref. [14] is based on the following observation done for conventional determinant

$$\det(\psi_{jk}) = \frac{1}{2} \text{tr}((\psi\epsilon)^T \epsilon\psi) \quad (25)$$

then using the following modifications ($SL(2)$ to $OSP(2|1)$) \oplus (transpose \rightarrow super-transpose) one can write

$$s\det(\psi_{XY}) = \frac{1}{2} \text{str}((\psi E)^{ST} E\psi), \quad (26)$$

what gives for two-superqubits explicitly

$$s\det(\psi_{XY}) = (\psi_{00}\psi_{11} - \psi_{01}\psi_{10} + \psi_{0\bullet}\psi_{1\bullet} + \psi_{\bullet 0}\psi_{\bullet 1}) - \frac{1}{2}\psi_{\bullet\bullet}^2 \quad (27)$$

This expression turned out to be good candidate for generalization of 2-tangle to the super 2-tangle

$$s\tau_{XY} = 4s\det\psi_{XY}(s\det\psi_{XY})^\# \quad (28)$$

It has basics property that $s\tau_{XY} = 0$ for product states. However its behaviour is different then we have used to for 2-tangle. Let us illustrate this using some examples:

Ex.1. The simple superstate [14]

$$|\phi\rangle = i|\bullet\bullet\rangle,$$

is maximally entangled in the sense of super 2-tangle: $s\tau_{XY} = 1$. This is an "algebraic artefact" because the separation of a superstate proportional to $|\bullet\bullet\rangle$ to product of one-superqubit states requires the nilpotent (a bodyless) coefficient in front of it being the product of two odd Grassmann numbers. Let us call such a state algebraically entangled.

Ex.2. Maximally super-entangled superstate [14]

$$|\psi\rangle = \frac{1}{\sqrt{3}}(|00\rangle + |11\rangle + i|\bullet\bullet\rangle) \quad (29)$$

$$s\tau_{XY}(|\psi\rangle) = 4\left(\frac{1}{3} + \frac{1}{2}\frac{1}{3}\right)^2 = 1,$$

Ex.3. similar superstate

$$|\psi\rangle = \frac{1}{\sqrt{3}}(|00\rangle + |11\rangle + |\bullet\bullet\rangle) \quad (30)$$

is not maximally entangled and $s\tau_{XY} = \frac{1}{9}$.

Ex.4. The following superstate

$$|\psi\rangle = \frac{1}{2}(|00\rangle + |11\rangle + \sqrt{2}|\bullet\bullet\rangle) = \frac{1}{\sqrt{2}}(\psi_{GHZ} + |\bullet\bullet\rangle) \quad (31)$$

has vanishing super 2-tangle, while it seems to be not separable (ψ_{GHZ} is the two qubit Greenberger Horne Zeilinger state).

Let us note that there exists generalization of the above construction to the three superqubits: $s\tau_{XYZ}$ given in the Ref. [14].

This approach is under development, recently there were proposed two-superqubit states that are "more nonlocal" than respective two-qubit states [17]. The analysis involves operation with the Grassmann valued probabilities and mentioned before choice of the way of ascribing final numerical values to such an objects - that always was a problematic issue of SUSY-QM in superHilbert spaces.

4 Squbit - reshaped superqubit

Soon after the introduction of the superqubit there was proposed its modification [15], with aims to cure the drawback of superqubits: the Grassmannian coordinates make difficult/impossible a probabilistic interpretation. The proposed modification to get rid of Grassmann valued probability amplitudes requires the introduction of auxiliary Grassmann variables θ_i in such a way that new superstate can be written in the form

$$|\psi\rangle = b|B\rangle + f_i\theta_i|F_i\rangle + b_{ij}\theta_i\theta_j|B_{ij}\rangle + f_{ijk}\theta_i\theta_j\theta_k|F_{ijk}\rangle + \dots \quad (32)$$

b, f are complex; $|B\rangle$ are even; $|F\rangle$ are odd. Such θ variables "neutralize" odd states $|F\rangle$. Then

$$\begin{aligned} \langle\langle\psi|\psi\rangle\rangle &\equiv \int e^{\sum_i \bar{\theta}_i \theta_i} \langle\psi|\psi\rangle \Pi_i d\bar{\theta}_i d\theta_i = \\ &|b|^2 + \sum |f_i|^2 + \sum_{i<j} |b_{ij}|^2 + \sum_{i<j<k} |f_{ijk}|^2 + \dots \end{aligned} \quad (33)$$

restores possibility of probabilistic interpretation. Let us note that it resembles earlier studied approach based on the commuting nilpotent η -variables.

Now, the superspace of superstates has the decomposition $\mathcal{H}_{BF} = \mathcal{H}_B \oplus \mathcal{H}_F$

- bosonic states: $|0\rangle, \theta_i\theta_j|0\rangle, \dots$
- fermionic states: $\theta_i|0\rangle, \theta_i\theta_j\theta_k|0\rangle, \dots$

where $|0\rangle$ is the Fock vacuum. Moreover one complements set of θ_i by $\bar{\theta}^j$ in such a way that $\theta_i, \bar{\theta}^j$ satisfy the Clifford algebra

$$\{\bar{\theta}^i, \theta_j\} = \delta_j^i, \quad \{\theta_i, \theta_j\} = 0, \quad \{\bar{\theta}^i, \bar{\theta}^j\} = 0 \quad (34)$$

and the new space $\mathcal{H}^\theta = \mathcal{H}_B^\theta \oplus \mathcal{H}_F^\theta$ is spanned on

$$\theta_i\theta_j \dots |0\rangle \otimes \theta_k\theta_l \dots |0\rangle = \theta_i\theta_j \dots \theta_k\theta_l \dots |0\rangle \quad (35)$$

To conclude the construction the extension of \mathcal{H}_{BF} to $\mathcal{H}_{BF} \otimes \mathcal{H}^\theta$ is proposed

$$\mathcal{H} = \left(\mathcal{H}_B \otimes \mathcal{H}_B^\theta \right) \oplus \left(\mathcal{H}_F \otimes \mathcal{H}_F^\theta \right) \quad (36)$$

giving the bosonic and fermionic dimensions of this space balanced i.e.

- 2^{N-1} of $|B_I\rangle$
- 2^{N-1} of $|F_I\rangle$

Finally superstate defining the squbit can be written in the following form

$$|\psi\rangle = b|0\rangle \otimes |B\rangle + f_i\theta_i|0\rangle \otimes |F_i\rangle + b_{ij}\theta_i\theta_j|0\rangle \otimes |B_{[ij]}\rangle + f_{ijk}\theta_i\theta_j\theta_k|0\rangle \otimes |F_{[ijk]}\rangle + \dots \quad (37)$$

What gives the simplest squbit:

$$|\psi\rangle = b|0\rangle \otimes |B\rangle + f\theta|0\rangle \otimes |F\rangle \quad (38)$$

Let us bring here the conclusion of the Authors of the Ref. [15]: *the auxiliary θ system allows the dynamical mixing between fermions and bosons of the \mathcal{H}_{BF} system, with a well-defined positive norm, necessary for a probabilistic interpretation.*

As it is seen from this brief account, the squbit has ballanced fermionic and bosonic degrees of freedom and is more close to usual supersymmetric models. It also seems to departure too far from the original superqubit model.

Final Comments

The notion of qubit is deeply rooted in quantum mechanics and the importance of this simple system was discovered after realizing the existence of new resource available in nature - quantum entanglement. For decades the real value of quantum entanglement was neglected and occasionally studied by people interested in quantum paradoxes and fundamental problems of the quantum mechanics. Presently it is intensively studied theoretically and its evidence is experimentally determined. Despite research based on conventional for quantum mechanics formalism of the Hilbert space, there are attempts to describe entanglement in different ways, using experiences from supermathematics and other algebraical structures. The η -Hilbert space formulation based on commuting nilpotent variables gives new functional tools to study many-qubit entanglement and to realize that many of interesting entangled states can be identified as elementary η -functions [16, 18]. On the other hand after four decades of supersymmetric theories there was proposed super-extension of the notion of qubit and super-entanglement of superstates with theoretically more nonlocality then in conventional many-qubit states. But physical meaning of this extension is still unclear.

References

- [1] Frydryszak A M, Int. J. Mod. Phys. **A 22** 2513 (2007)

- [2] Mandilara A, Akulin V M, Smilga A V, Viola L, Phys. Rev. A **74**, 022331 (2006)
- [3] Ziegler K, Europhys. Lett. **9** 277 (1989)
- [4] Palumbo F, Phys. Rev. **D** 50 2826 (1994)
- [5] Frydryszak A M, Int. J. Mod. Phys. **A** 22 2513 (2007) [[arXiv:hep-th/0609072](#)]
- [6] Frydryszak A M, Int. J. Mod. Phys. **A25** 951 (2010)
- [7] Hruby J, *Supersymmetry and qubit field theory*, [arXiv:quant-ph/0402188](#)
- [8] DeWitt B, *Supermanifolds* CUP, Cambridge, 1984. 2nd ed.
- [9] Frydryszak A, Jakobczyk L, Lett. Math. Phys. **16** 101 (1988)
- [10] Frydryszak A, Lett. Math. Phys. **26** 105 (1992)
- [11] Rogers A, J. Math. Phys. **21** 1352 (1980)
- [12] Rogers A, *Supermanifolds: Theory and Applications*, World Scientific, Singapore, 2007
- [13] Nicolai H, J. Theor. Phys A: Math. Gen. **9** 1497 (1976)
- [14] Borsten L, Dahanayake D, Duff M J, Rubens W, Phys.Rev. **D** 81 105023 (2010)
- [15] Castellani L, Grassi P A, Sommovigo L, *Quantum computing with superqubits* [arXiv:1001.3753](#)
- [16] Frydryszak A M, *Nilpotent quantum mechanics, qubits, and flavors of entanglement* [arXiv:0810.3016](#)
- [17] Borsten L, Bradler K, Duff M J, *Tsirelson's bound and supersymmetric entangled states* [arXiv:1206.6934](#)
- [18] A. M. Frydryszak, Int. J. Geom. Meth. Mod. Phys. **09** 1261005(2012).